# 2.1 Introduction to Equations

In this chapter we will begin to look at a topic that will be revisited throughout the entire rest of the course. That is, we want to talk about equations.

We start with the following definition and example.

| **<u>Definition</u>**: **<u>Equation</u>**- a statement of equality of two algebraic expressions.<br>For example,<br>$4x - 5 = 7x + 2$<br>Left side      Right side |
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The difference between an equation and the expressions that we discussed in chapter 1, is that an equation has its own equal sign built into it. This means that they have different rules to follow than expressions do. We will discuss the rules later in this chapter and later in this textbook.

So, the question is, what do we want to do with equations?

We want to find the values for the variables the make the equation a true statement. We call these **<u>solutions</u>** and the process we use to find the solutions is called **<u>solving</u>**.

Before we get to the process of solving an equation, the first thing we want to do is be able to check to see if a given value is, in fact, a solution to an equation.

We have the following steps to do this.

| **<u>To Check a Solution of an Equation</u>** |
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| 1. Substitute the given value into the original equation.<br>2. Perform the order of operations on each side.<br>3. If step 2 makes a true statement, then the value is a solution. If not, it's not. |

<u>Example 1</u>:

a. Is $y = 4$ a solution to $3y - 4 = 2y$?

b. Is $n = 2$ a solution to $7 - 3n = 2$?

Solution:

a. To check to see if $y = 4$ is a solution, we need to start with substituting the 4 into the equation where the y is. We then work out each side it see if we get a true statement. We have

$$3y - 4 = 2y$$
$$3(4) - 4 = 2(4)$$
$$12 - 4 = 8$$
$$8 = 8$$

Since we got a true statement, $y = 4$ is a solution to the equation.

b. Like part a above, we substitute 2 into the equation for the n, and see if we get a true statement. We have

$$7 - 3n = 2$$
$$7 - 3(2) = 2$$
$$7 - 6 = 2$$
$$1 = 2$$

Since we do not have a true statement, 2 is not a solution to the equation.

Even though checking a possible solution to an equation is important, it is far more important to be able to start with an equation and generate the solution.

To do this, we need to know the kinds of things we can do to an equation. We have to following list of properties of equations.

| **Properties of Equations** |
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| Add/Subtract Property:<br>If a = b, then a + c = b + c and a – c = b – c .<br>i.e. We can add or subtract anything on both sides of an equation.<br><br>Multiply/Divide Property:<br>If a = b, then ac = bc and a/c = b/c.<br>i.e. We can multiply or divide anything on both sides of an equation<br><br>Symmetric Property:<br>If a = b, then b = a. |

With these properties we will be able to solve equations. The goal is solving an equation is to get the equation to look like

"**variable = number**"

Once we have done this, the variable equals the number. So whatever number that is, would be the solution to the equation.

Let's start looking at some examples of solving equations. We will start with the very simple, and by the end of the next section, we will solve equations that are fairly complex.

Example 2:

Solve.

a. $x – 7 = 11$ b. $-y – 6 = -2y + 1$ c. $5(x + 2) = 4(x + 1)$

Solution:

a. To solve the equation. We need to try to get the variable, x all alone on one side of the equation. In this case, we simply need to get the -7 away from the left side.

   To do this, we can use the add/subtract property to add 7 to both sides. This will eliminate the 7 from the left, and move it to the right side of the equation.
   We get

$$\begin{array}{rcl} x - 7 &=& 11 \\ +7 && +7 \\ \\ x &=& 18 \end{array}$$

   Since we now have "variable = number", the equation is solved. So the solution to the equation is x = 18. When solving equations, we always put the solution into what is called a **solution set**. This means our solution set is {18}.

b. This time, the equation is much more challenging. Here, we will need to collect all of the variable terms on one side of the equation and all of the constant terms on the other.
   We again use the add/subtract property to accomplish this.
   We proceed as follows

$$-y - 6 = -2y + 1$$
$$+2y \qquad +2y$$ Add 2y to both sides

$$y - 6 = 1$$
$$+6 \quad +6$$ Add 6 to both sides

$$y = 7$$

So the equation now looks like “variable = number” so the solution set is {7}.

c. Lastly, we need to start this time with distributing the 5 and the 4 through each side and then get the variables on one side, numbers on the other, like we did in part b. We get

$$5(x + 2) = 4(x + 1)$$ Distribute to get rid of the ( )

$$5x + 10 = 4x + 4$$
$$-4x \qquad -4x$$ Subtract 4x from each side

$$x + 10 = 4$$
$$-10 \quad -10$$ Subtract 10 from each side

$$x = -6$$

Since the variable is all alone on one side, the equation is solved. Our solution set is {-6}.

The equations in example 2 only used the “Add/Subtract Property”. In the next example, we will take a look at the “Multiply/Divide Property”.

<u>Example 3</u>:

Solve.

a. $3a = 18$ b. $\frac{2}{5}x = 10$ c. $\frac{2}{9}x = \frac{1}{3}$

Solution:

a. In this example, we need to, again, get the variable all alone on one side. That means, we need to get rid of the 3 in front of the a. Since the 3 is multiplied by the a, we can divide both sides by 3.
We get

$$\frac{3a}{3} = \frac{18}{3}$$

$$a = 6$$

So the solution set is {6}.

b. This time, we have a fraction involved. There are a number of different ways to deal with a fraction in an equation. The best way is to use a process called “clearing fractions”.

The idea behind clearing fractions is that we can get rid of all of the fractions in the equation by multiplying both sides by the LCD.

Once the fractions are gone, we solve as usual.

In this case, the LCD is 5. So multiplying we get

$$\frac{2}{5}x = 10$$

$$5 \cdot \left(\frac{2}{5}x\right) = (10) \cdot 5$$ Multiply by the LCD,5

$$2x = 50$$ Reduce

$$\frac{2x}{2} = \frac{50}{2}$$ Divide by 2 to get x alone

$$x = 25$$

So the solution set is {25}.

c. Again, we will clear fractions by multiplying both sides by the LCD of 9.
We get

$$\frac{2}{9}x = \frac{1}{3}$$

$$9 \cdot \left(\frac{2}{9}x\right) = \left(\frac{1}{3}\right) \cdot 9$$ Multiply by the LCD,9

$$2x = 3$$ Reduce

$$\frac{2x}{2} = \frac{3}{2}$$ Divide by 2 to get x alone

$$x = \frac{3}{2}$$

So the solution set is $\left\{\frac{3}{2}\right\}$.

Now, let's put it all together and solve some equations that use both properties.

<u>Example 4</u>:

Solve.

a. 5x – 2 = 13 b. 4x – 1 = 6x + 2

Solution:

a. To get the x all alone on one side of the equation, the first thing we should do is get the term containing the x alone on one side. Then it will be fairly simple to solve the rest of the equation. We get

5x - 2 = 13
+2 +2 Add 2 to both sides

$\frac{5x}{5} = \frac{15}{5}$ Divide by 5 to get x alone

x = 3

So the solution set is {3}.

b. Again, we want to get the term containing the variable alone on one side. However, we need to start by getting all of the variables to the same side. Then we can get the term alone, and solve as usual. We have

$$\begin{aligned} 4x - 1 &= 6x + 2 \\ -6x \quad & \quad -6x \end{aligned}$$

$$\begin{aligned} -2x - 1 &= 2 \\ +1 \quad & \quad +1 \end{aligned}$$

$$\frac{-2x}{-2} = \frac{3}{-2}$$

$$x = -\frac{3}{2}$$

So the solution set is $\left\{-\frac{3}{2}\right\}$.

Now that we can solve some equations, we want to be able to apply this knowledge. We will look at application in depth in a later section, but for now, we want to take a look at a simple type of application... percents.

| **The Percent Equation** |
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| $$P \cdot B = A$$ Where, P is the percent, written usually as a decimal, B is the base and generally follows the word "of", and A is the amount. |

When we say the "base follow the word of" we mean when the problems is written in, what is called a "what-is-of" statement.

Let's take a look at some of these in our next example.

Example 5:

a. What percent of 8 is 0.5?

b. 45% of what is 9?

Solution:

a. To do the simple percent question, we simply need to identify our percent, base and amount.

Since the problem is already in a "what-is-of" statement, the percent will stay at "P" because that is what we are looking for, the base follows the "of". So the base is 8, and the amount must, therefore, be 0.5.

Now, putting them into the percent equation gives us

$$P \cdot B = A$$

$$P \cdot 8 = 0.5$$

$$\frac{P \cdot 8}{8} = \frac{0.5}{8} \qquad \text{Solve by dividing by 8}$$

$$P = 0.0625$$

This is the decimal version of our percent answer. Therefore, the answer is 6.25%

b. Again, to solve, we need to identify each part of the percent equation. Here, the percent is 45%. This means our "P" is 0.45 (since we must use it as a decimal in the percent equation), the base is missing (since after the word "of" is the word "what") and the amount must be 9.

So putting these values in we get

$$P \cdot B = A$$

$$0.45 \cdot B = 9$$

$$\frac{0.45 \cdot B}{0.45} = \frac{9}{0.45}$$

$$B = 20$$

So our answer is 20.

Once we can do the simple percent question, we can extend to the more complex percent question. Even though it seems much more difficult, it all comes back to the percent equation.

<u>Example 6</u>:

A down payment of $4500 was paid on a new car. This is 15% of the cost of the car. What is the cost of the car?

Solution:

On a more substantial percent problem, we should start by changing the equation into a simple "what-is-of" statement so that we can solve it from there.

In this case, the second equation gives us all that we need to know. We can translate the statement into the "what-is-of" statement by simply inserting the 4500 in for the word "This".

We get, "4500 is 15% of what?"

Now we solve as we did in example 5.

$$P \cdot B = A$$

$$0.15 \cdot B = 4500$$

$$\frac{0.15 \cdot B}{0.15} = \frac{4500}{0.15}$$

$$B = 30000$$

So the car cost $30,000.

<u>Example 7</u>:

Donal's Auto Repair has its regularly priced $125 tune up on sale for 16% off. What is the sale price?

Solution:

Finally, we again want to convert the problem into a simple “what-is-of” statement. Since the tune up is 16% off, we want to know “What is 16% of 125?” This is our statement.

So the percent equation gives

$$P \cdot B = A$$

$$0.16 \cdot 125 = A$$

$$20 = A$$

As it turns out, this is the amount to be taken off. So the actual sale price is \$125 - \$20 which is \$105.

## 2.1 Exercises

1. Is x = 4 a solution to 2x + 9 = 3?
2. Is x = -2 a solution to 5x + 7 = 12?
3. Is x = 3 a solution to 3x - 2 = x + 4?
4. Is x = 1 a solution to 5x + 4 = 1 + 2x?
5. Is x = -1 a solution to 2x(x - 1) = 3 - x?
6. Is x = 0 a solution to x(x - 1) = x?
7. Is x = -4 a solution to x(x + 4) = $x^2$ + 16 ?
8. Is x = 2 a solution to x(x - 2) = $x^2$ - 4?
9. Is x = ¼ a solution to 2x - 3 = 1 - 14x?
10. Is x = ½ a solution to 3x + 6 = 4?

Solve.

11. $x - 6 = 12$
12. $x + 4 = 23$
13. $y + 16 = 42$
14. $y - 3 = 0$
15. $a - 8 = 13$
16. $b + 7 = 56$
17. $z - 45 = 0$
18. $n - 5 = 91$
19. $4x - 4 = 3x + 2$
20. $5x - 18 = 4x + 7$
21. $-y + 4 = -2y + 8$
22. $3a + 4 = 2a - 8$
23. $1 + 10z = 9z + 4$
24. $7x = 6x + 1$
25. $3(x - 4) = 2(x - 5)$
26. $2(x - 3) = x - 4$
27. $12(y - 3) = 11(y + 3)$
28. $3(a - 4) = 2(a - 1)$
29. $4(b + 2) = 3(b + 1)$
30. $7(c - 4) = 6(c + 3)$
31. $6(n - 3) = 5n$
32. $4(x + 1) = 3x$
33. $8x = 7(x + 1)$
34. $3p = 2(p - 8)$
35. $3(w - 1) = 2w - 3$
36. $7(u + 2) = 6u + 14$
37. $3x = 9$
38. $4x = 12$
39. $-6y = 18$
40. $-7y = -7$
41. $-10a = 15$
42. $8y = 4$
43. $21b = 0$
44. $15x = 0$
45. $\frac{x}{2} = -4$
46. $\frac{3}{4}x = -9$
47. $\frac{2}{5}a = -16$
48. $\frac{7}{2}x = 21$
49. $\frac{1}{3}x = \frac{5}{9}$
50. $\frac{1}{5}x = \frac{2}{15}$
51. $-\frac{3}{7}a = \frac{4}{3}$
52. $\frac{4}{5}n = -\frac{5}{6}$
53. $2x - 1 = 3$
54. $3x + 1 = 7$
55. $-4x + 3 = 15$
56. $-2x + 7 = 13$

57. $-2y + 8 = 7$

58. $6y - 6 = 17$

59. $-3a + 4 = -11$

60. $-6n + 12 = -9$

61. $x + 2 = 6x - 8$

62. $5x - 2 = 2x + 1$

63. $2a - 15 = 8a + 7$

65. $9b + 3 = 12b - 15$

66. $4d - 12 = 12d - 4$

67. $7(x - 5) = 9(x - 3)$

68. $3(x - 5) = 12(x - 4)$

69. $3(2x + 7) = 12(2x - 4)$

70. $5(3x - 3) = 6(2x + 2)$

71. What is 18% of 150?

72. 52% if 95 is what?

73. 126 is 84% of what?

74. 38% of what is 171?

75. What percent of 1500 is 693?

76. What percent of 80 is 20?

77. 325% of 4.4 is what?

78. What is 112% of 5?

79. 160% of what is 40?

80. 1 is 0.25% of what?

81. 750 is what percent of 50?

82. What percent of 11 is 88?

83. What is 87.4% of 255?

84. 5.3 is what percent of 50?

85. 14 is 175% of what?

86. 10.4% of what is 52?

87. Jennifer buys a $125 camera. Sales tax on the camera is 9.5%. How much will she pay in sales tax?

88. If Tim gets 27 problems correct on a test with 45 problems. What is Tim's percent correct?

89. A Verizon dealer found that 86% of the 250 phones that were sold during one month were Iphones. How many Iphones were sold in the month?

90. A truck retail sales company made 4.5% profit on sales of $3,600,000. What is the company's profit?

91. In a recent marathon, 23,520 runners crossed the finish line. This is 98% of the runners who started the marathon. How many runners started the marathon?

92. A basketball team lost 14 of its 56 games they played this season. What percent of the games played did the team win?

93. An online survey asked how people planned to use their tax refunds. 748 people said they would save the money. This represents 22% of the total number of people who took the survey. How many people took the survey?

94. Sports Chalet has its regularly priced $160 sleeping bag on sale for $120. What is the discount rate?

95. Toyota increased the average mileage on its line of trucks from 17.5 miles per gallon to 18.2 miles per gallon. Find the percent increase in mileage.

96. During 1 year, the number of people subscribing to DirectTV increased by 87%. If the number of subscribers at the beginning of the year was 2.3 million, how many subscribers were there at the end of the year?

97. Jen's Camera Shop sells its Model JB camera for $115.50. It only costs the store $70 when they buy it wholesale. What is markup rate at Jen's Camera Shop?

98. Roger Dunn's Golf Shop uses a markup rate of 45% on a set of Tour Pro golf clubs that costs the shop $210. What is the selling price?

99. Target employs 1200 people during the holiday season. At the end of the season, the store reduces the number of employees by 45%. How many employees will be employed at the end of the holiday season?